Killara HIGH SCHOOL

Student Number

2016

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time 3 hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- A formula sheet is provided with this paper
- In Questions 11-16 show relevant mathematical reasoning and/or calculations
- Start a new booklet for each question

Total Marks – 100

Section I - Pages 1 - 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - Pages 7 - 15

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hour and 45 minutes for this section

Question	Marks
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
15	/15
16	/15
Total	/100

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 40% of the Higher School Certificate Course Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for questions 1 – 10 (Detach from paper)

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1) Cards numbered form one to fifteen are placed in a bag. One card is selected at random from the bag. What is the probability that the number is an odd number or a number divisible by 3?

(A) $\frac{13}{15}$

(B) $\frac{2}{3}$

(C) $\frac{1}{5}$

(D) $\frac{8}{225}$

2) The solutions to the equation $7x^2 + 3x - 4 = 0$ are:

(A) $x = 1$ and $x = -4$

(B) $x = -1$ and $x = 4$

(C) $x = -1$ and $x = \frac{4}{7}$

(D) $x = 1$ and $x = -\frac{4}{7}$

3) A particle is has an initial displacement of -8m. The particle is moving at $-3ms^{-1}$ with an acceleration of $2ms^{-2}$. Which of the following is correct:

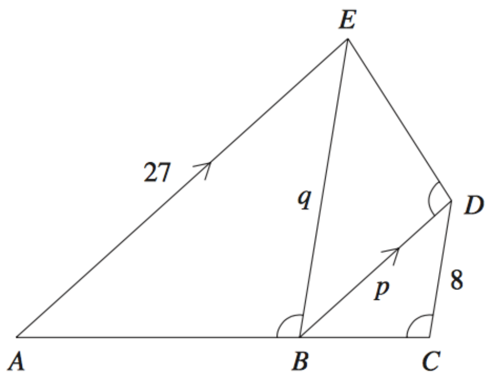
(A) the particle is moving to the left and slowing down

(B) the particle is moving to the left and getting faster

(C) the particle is moving to the right and slowing down

(D) the particle is moving to the right and getting faster.

4) In the diagram, AE is parallel to BD, $AE = 27$, $CD = 8$, $BD = p$ and $BE = q$.



ΔABE is similar to ΔBCD.
Which of the following is then true?

(A) $\dfrac{27}{p} = \dfrac{q}{BC}$

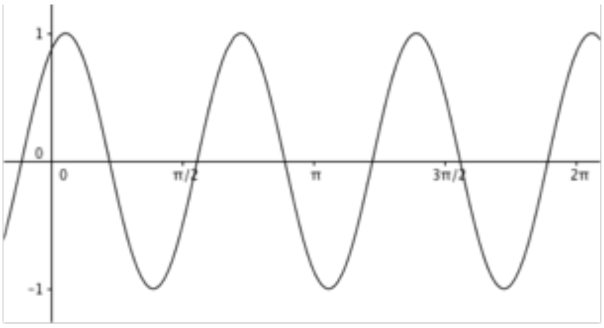
(B) $\dfrac{27}{8} = \dfrac{q}{p}$

(C) $\dfrac{27}{q} = \dfrac{p}{8}$

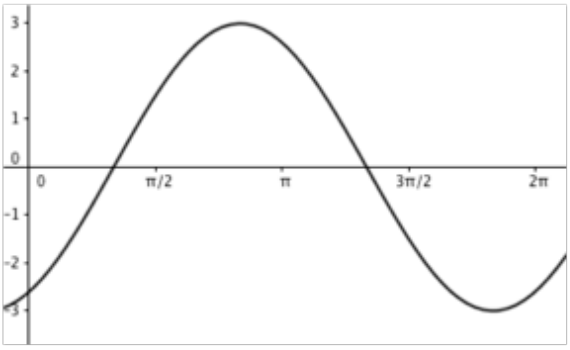
(D) $\dfrac{27}{p} = \dfrac{8}{q}$

5) Which graph below matches the equation $f(x) = 3\sin\left(x - \frac{\pi}{3}\right)$?

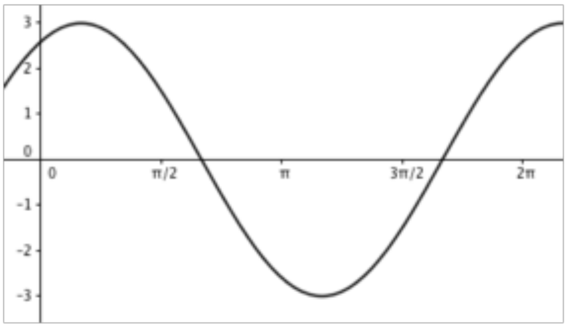
A



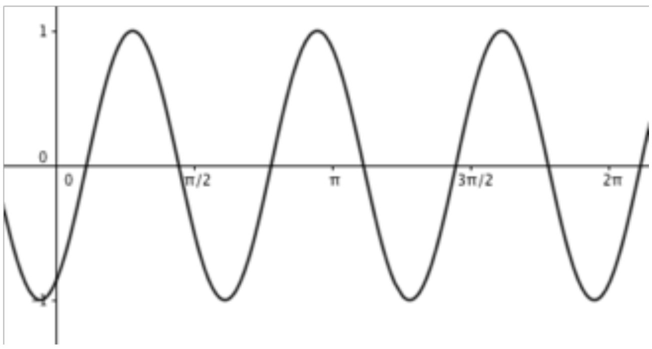
B



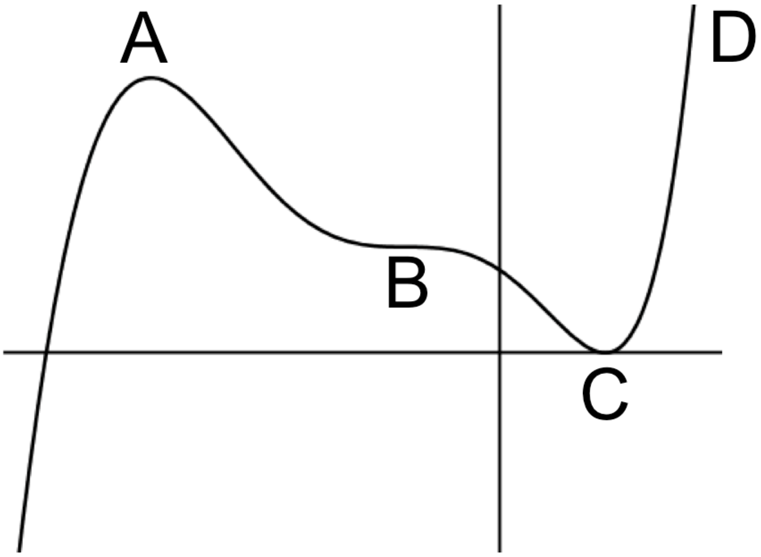
C



D



6) At which point on the graph of $f(x)$ shown below is $f'(x) < 0$ and $f''(x) = 0$?



(A) A

(B) B

(C) C

(D) D

7) Which are the solutions of $\tan^2 x = 1$ for $0 \le x \le 2\pi$?

(A) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(B) $\frac{\pi}{4}, \frac{5\pi}{4}$

(C) $\frac{3\pi}{4}, \frac{7\pi}{4}$

(D) $\frac{-3\pi}{4}, \frac{-7\pi}{4}$

8) $3^{\frac{5}{4}} \times 5^{\frac{3}{4}}$ is equal to:

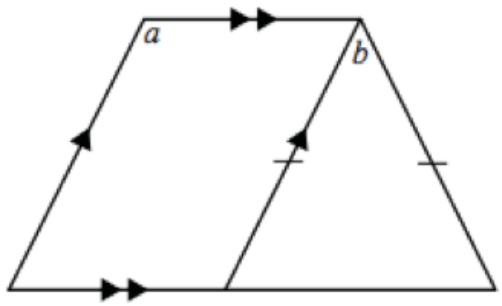
(A) $9 \times 5^{\frac{3}{4}}$

(B) $15 \times 5^{\frac{3}{4}}$

(C) $3^{\frac{1}{2}} \times 15^{\frac{3}{4}}$

(D) $15 \times 15^{\frac{3}{4}}$

9) A composite shape is made up of a parallelogram and a triangle as shown below.



Which of the following is always true?

(A) $b = 2a$

(B) $a = 2b$

(C) $a + b = 180$

(D) $2a - b = 180$

10) If Town A is due west of town B, and Town C is due south of Town B, then the bearing of Town A from Town C is:

(A) Between 0° and 90°

(B) Between 90° and 180°

(C) Between 180° and 270°

(D) Between 270° and 360°

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) The Earth has a diameter of 12742 km. Write this number in scientific notation, correct to two significant figures **2**

(b) Differentiate $\frac{5}{x^3} - 3x^7$ **1**

(c) Solve $\frac{7-x}{3} - \frac{4+3x}{4} = -1$ **3**

(d) Find integers a and b such that $\frac{4}{3-\sqrt{7}} = a + b\sqrt{7}$ **2**

(e) A card is drawn from a standard packet of 52 cards. **2**

What is the probability that the card drawn is a Club or the Jack of Hearts?

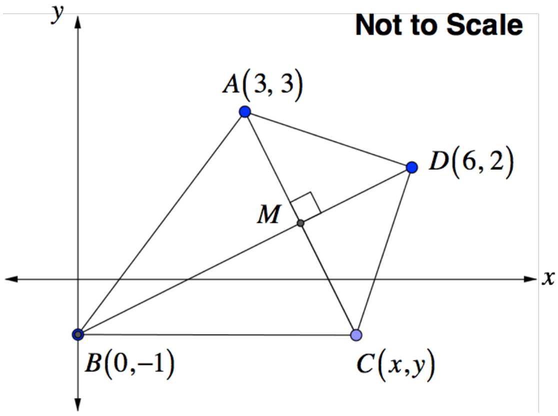
(f) Evaluate giving your answer as an exact value **2**

$$\int_0^{\frac{\pi}{3}} \cos\left(\frac{x}{2}\right) dx$$

(g) Differentiate $y = \frac{3x}{x^2+3}$ giving your answer in simplified form **3**

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) In the quadrilateral *ABCD*, the points *A*, *B,* and *D* are (3, 3), (0, -1) and (6,2) respectively. The line *BD* bisects the line *AC* at right angles to the point *M*.



i) Find the distance *BD* **1**

ii) Show that the gradient of *BD* is $\frac{1}{2}$ **1**

iii) Show that the equation of the line *BD* is $x-2y-2=0$ **1**

iv) Show that the equation of the line *AC* is $2x+y-9=0$ **1**

v) Find the coordinates of *M* **2**

vi) Hence, or otherwise find the coordinates of *C* **1**

(b) Prove that $$\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}=1-\sin\theta\cos\theta$$ **2**

(c) If α and β are the roots of $2x^2-5x-7=0$ find the value of $\alpha^2+\beta^2$ **2**

(d) Differentiate with respect to *x*:

i) $x^4\sin 2x$ **1**

ii) $(3+\log_e x)^5$ **1**

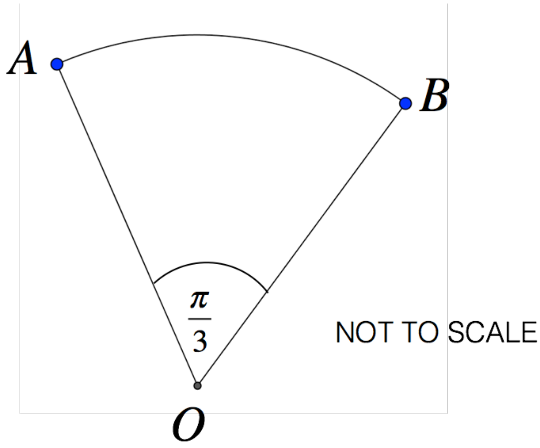
Question 13 (15 marks) Use a SEPARATE writing booklet

(a) Cinca Island is 150km due east of port A. A boat is 100km due south of Cinca Island. Calculate the bearing of the boat from port A to the nearest degree. **2**

(b) AOB is a sector of a circle, centre O, with angle $\frac{\pi}{3}$. **2**

The area of the sector AOB is $\frac{25\pi}{6} cm^2$.

What is the exact length of the arc AB ?



Question 13 continues on page 10

Question 13 (continued)

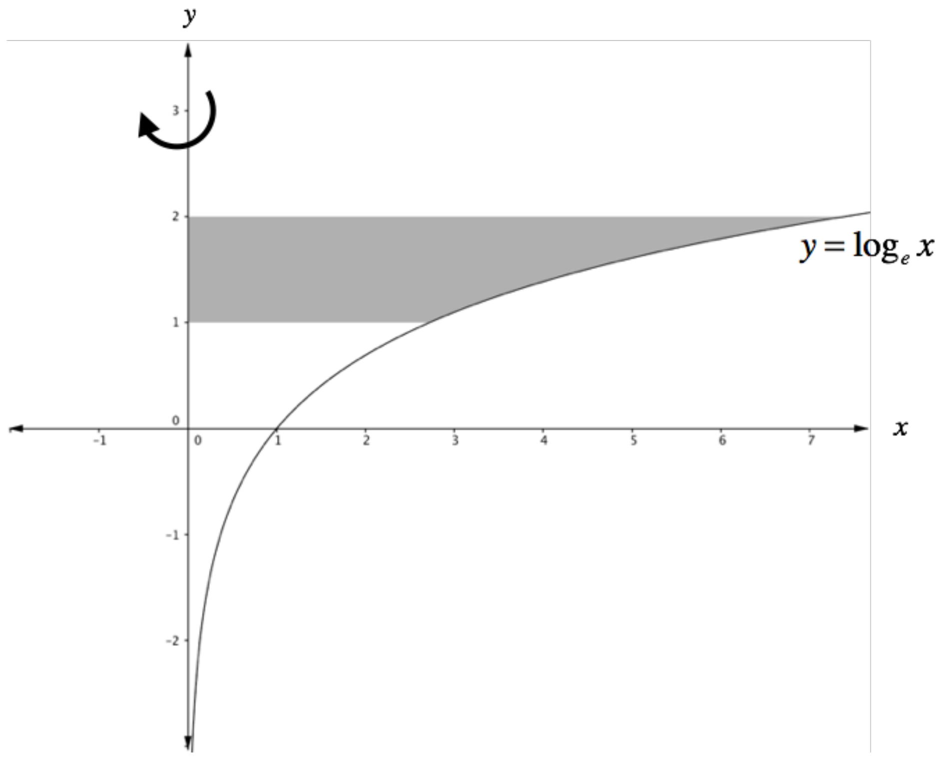
(c) Consider the function $f(x) = 2x^3 - 4x^2$

i) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature. **3**

ii) Find the point of inflexion. **2**

iii) Sketch the curve showing where it meets the axes. **2**

(d) The curve $y = \log_e x$ is shown below.



i) Show that when x is made the subject of the equation, the equation of the curve is $x = e^y$. **1**

ii) In the diagram the shaded region is bounded by the curve $y = \log_e x$, the y-axis and the lines $y = 1$ and $y = 2$ **3**

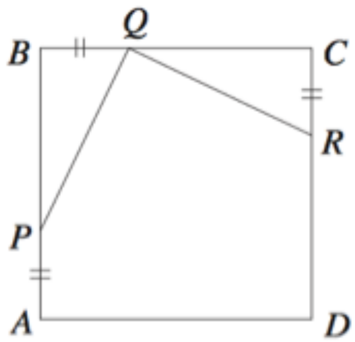
The shaded region is rotated about the y-axis.

Calculate the exact volume of the solid of revolution formed.

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Solve the following: $2^{2x} - 12\left(2^{x}\right) + 32 = 0$ **3**

(b) *ABCD* is a square. The points *P, Q* and *R* lie on *AB, BC* and *CD* respectively so that *AP=BQ=CR*.



i) Prove that $\Delta PBQ \equiv \Delta QCR$. **2**

ii) Prove that *PQ* is perpendicular to QR. **2**

(c) A particle moving in a straight line is initially at the origin. The displacement, in metres, after t seconds is given by $x = 2t - 3\log_e(t+1)$.

i) Find an expression for the velocity. **2**

ii) Find the initial velocity. **1**

iii) Find when the particle is at rest and its position at this time (answer correct to 2 decimal places) **2**

(d) There are five candidates, Allan, Brown, Chin, Davis and Echert standing for the seat of Bradfield in the federal election. Their names are written on pieces of paper and randomly drawn from a barrel to determine their positions on the ballot paper. The candidate picked first goes at the top of the list

i) What is the probability that Davis is drawn first? **1**

ii) What is the probability that the order the names appear on the ballot paper is as follows. **2**

Allan	
Brown	
Chin	
Davis	
Echert	

Question 15 (15 marks) Use a SEPARATE writing booklet

(a) Crumponium is a rare radioactive substance that decays with a highly toxic residue. The rate of change is given by

$$\frac{dM}{dt} = -kM$$

where k is a positive constant and M is the mass present.

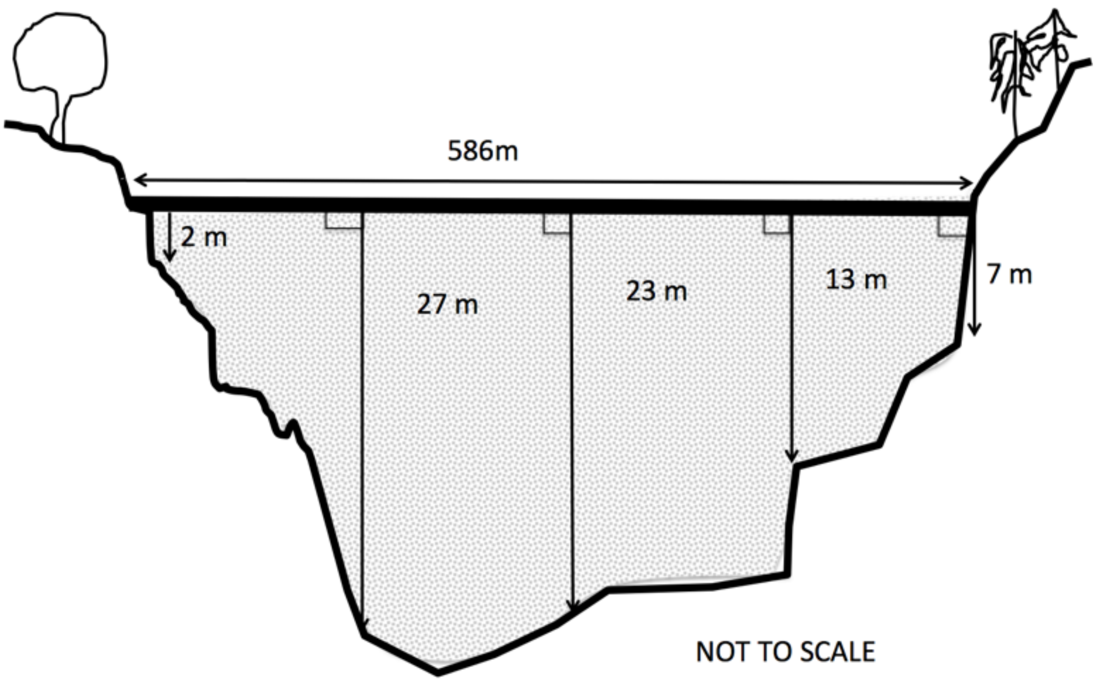
i) The half-life of Crumponium is 29 years. This means it takes 29 years for 100g to decay to 50g. Find the value of k correct to 3 significant figures. **2**

ii) A decaying bag of Crumponium is found illegally dumped at a landfill site. It is weighed and its mass is 12 kg. Calculate the original mass if it was dumped 10 years ago. Give your answer to 2 decimal places. **1**

(b) Engineers have recorded the average depth measurements for a new dam at Jindabyne.

The diagram below shows the cross sectional area along the dam wall. The depth measurements are taken at equidistant points along the dam wall. The diagram is not to scale.

Use Simpsons Rule to determine the cross sectional area of the dam wall. Give your answer to the nearest square metre. **2**



Question 15 continues on page 13

(c) For what values of k will the expression $kx^2 + 2x + k$ always be negative? **2**

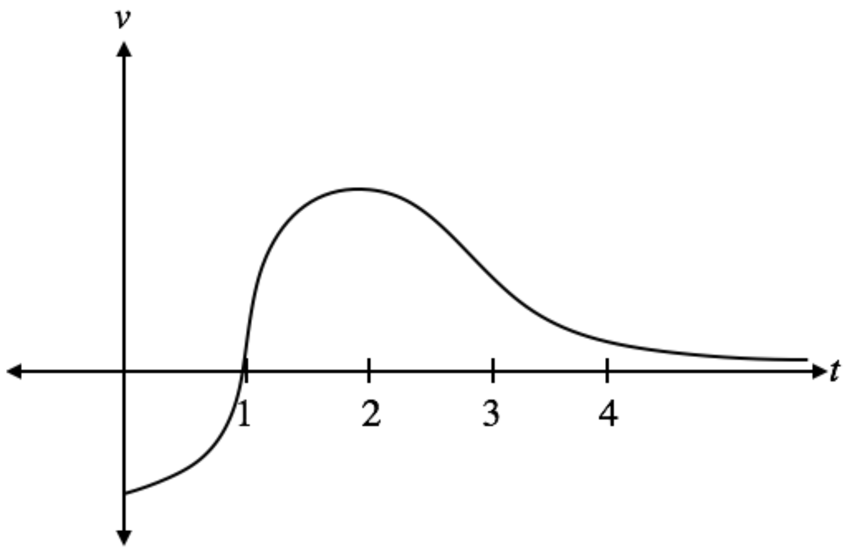
(d) Show that the locus of a point that moves so that its distance from the point $A(-5, 2)$ is twice its distance from the point B $(1, 2)$ is a circle with centre $(3, 2)$ and Radius $r = 4$ **3**

(e) A circular barbeque plate is being heated so the rate of increase of the area $A\,cm^2$ after t minutes is given by **3**

$$\frac{dA}{dt} = \frac{\pi}{10(t+1)}$$

The plate has an initial Area of 45 cm^2. Find the area of the plate after it has been heated for 50 minutes (give your answer correct to 2 decimal places)

(f) A particle moves along the x-axis. Its velocity v ms^{-1} after t seconds is shown in the diagram

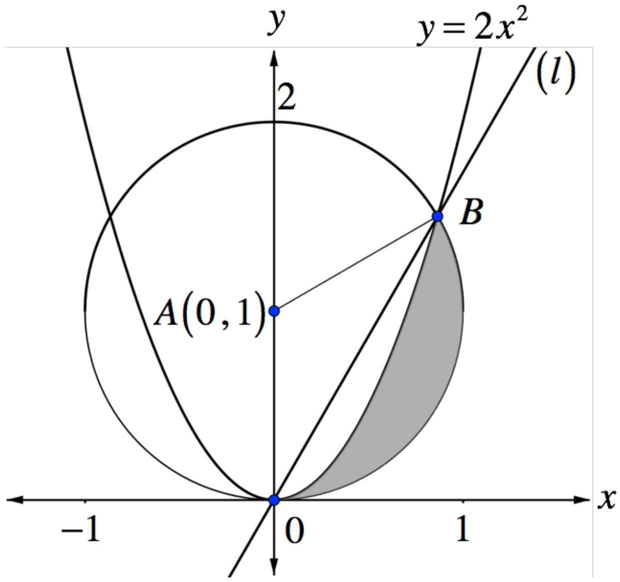


i) Initially is the particle moving to the left or the right. Explain your answer. **1**

ii) How would you calculate the distance the particle travels in the first 3 seconds? **1**

Question 16 (15 marks) Use a SEPARATE writing booklet

(a) The circle centred at A (0, 1) and with radius 1 unit intersects the parabola $y = 2x^2$ at the origin O and the point B. The line l passes through O and B as shown in the diagram



i) Show that the coordinates of B are $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$ **2**

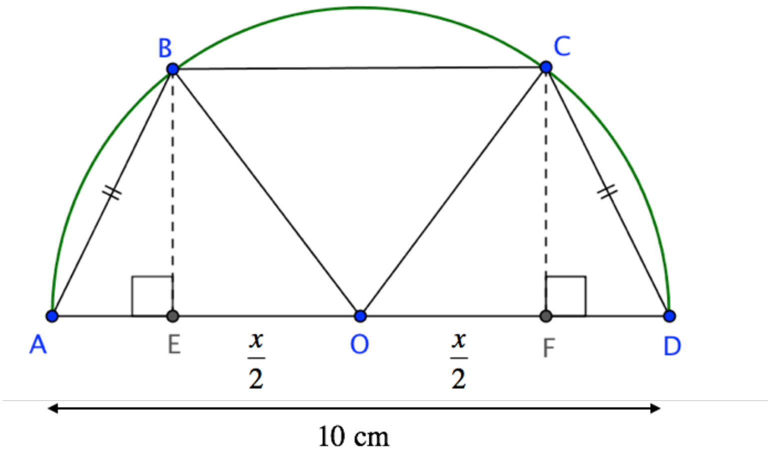
ii) Find the angle OB makes with the positive x axis **1**

iii) Show that $\angle OAB = \frac{2\pi}{3}$ **1**

iv) Find the shaded area bounded by the circle and the parabola in the first quadrant as shown in the diagram **3**

Question 16 continues on page 15

(b) An isosceles trapezium *ABCD* is drawn with its vertices on a semicircle centre *O* and diameter 10 cm.



i) If $EO = OF = \frac{x}{2}$ show that $BE = \frac{1}{2}\sqrt{100 - x^2}$ **2**

ii) Show that the area of the trapezium *ABCD* is given by: **2**

$$A = \frac{1}{4}(x + 10)\sqrt{100 - x^2}$$

iii) Hence find the length of *BC* so that the area of the trapezium is a maximum **4**

End of Examination ☺

	Year 12 Trial Solutions
	Multiple Choice answers 1 B 2 C 3 A 4 C 5 B 6 B 7 A 8 C 9 D 10 D

Question 11

(a) The Earth has a diameter of 12742 km. Write this number in scientific notation, correct to two significant figures.

2 marks	Writes number in scientific notation rounded correctly to two significant figures
1 mark	Writes number in scientific notation

$$\begin{aligned} 12\,742 &= 1.2742 \times 10^4 \\ &= 1.3 \times 10^4 \end{aligned}$$

(b) Differentiate $\dfrac{5}{x^3} - 3x^7$

1 mark	Correct Derivative

$$\begin{aligned} y &= \frac{5}{x^3} - 3x^7 = 5x^{-3} - 3x^7 \\ \therefore y' &= -15x^{-4} - 21x^6 \end{aligned}$$

(c) Solve $\dfrac{7-x}{3} - \dfrac{4+3x}{4} = -1$

3 marks	Obtains correct solution (3) with working
2 marks	Multiplies by common denominator and expands brackets or similar to obtain (2)
1 mark	Multiplies by common denominator or similar to obtain (1)

$$\begin{aligned} \frac{7-x}{3} - \frac{4+3x}{4} &= -1 \\ \therefore 4(7-x) - 3(4+3x) &= -12 && (1) \\ \therefore 28 - 4x - 12 - 9x &= -12 && (2) \\ \therefore -13x &= -28 \\ \therefore x &= -\frac{28}{13} && (3) \end{aligned}$$

Question 11 continued

(d) Find integers a and b such that $\dfrac{4}{3-\sqrt{7}} = a + b\sqrt{7}$

2 marks	Obtains correct values of a and b with working
1 mark	Multiplies by conjugate of denominator $3+\sqrt{7}$

$$\frac{4}{3-\sqrt{7}} \quad \frac{\times(3+\sqrt{7})}{\times(3+\sqrt{7})} = \frac{12+4\sqrt{7}}{9-7}$$
$$= \frac{12+4\sqrt{7}}{2}$$
$$= 6+2\sqrt{7} = a+b\sqrt{7}$$
$$\therefore a = 6 \quad b = 2$$

(e) A card is drawn from a standard pack f 52 cards.
What is the probability that the card drawn is a Club or the Jack of Hearts?

2 marks	Finds correct probability (no mark deducted if not simplified)
1 mark	States the P(club) **A**ND the P(Jack of Hearts)

$\text{P(club)} = \dfrac{1}{4} \qquad \text{P(Jack of Hearts)} = \dfrac{1}{52}$

$$\therefore \text{P(Club OR Jack of Hearts)} = \frac{1}{4} + \frac{1}{52} \qquad \text{(since Mutually Exclusive)}$$
$$= \frac{14}{52}$$
$$= \frac{7}{26}$$

(f) Evaluate giving your answer as an exact value $\displaystyle\int_0^{\frac{\pi}{3}} \cos\left(\frac{x}{2}\right)dx$

2 marks	Substitutes in bounds correctly and simplifies to obtain correct result
1 mark	Finds the correct primitive function $\left[2\sin\left(\frac{x}{2}\right)\right]$

$$\int_0^{\frac{\pi}{3}} \cos\left(\frac{x}{2}\right)dx = \left[2\sin\left(\frac{x}{2}\right)\right]_0^{\frac{\pi}{3}}$$
$$= 2\sin\frac{\pi}{6} - 2\sin 0$$
$$= 1 - 0$$
$$= 1$$

Question 11 continued

(g) Dfferentiate $y = \dfrac{3x}{x^2+3}$ giving your answer in simplified form

3 marks	Correct answer with full working simplified
2 marks	Correct substitution into quotient rule
1 mark	Identifies quotient rule parts correctly)

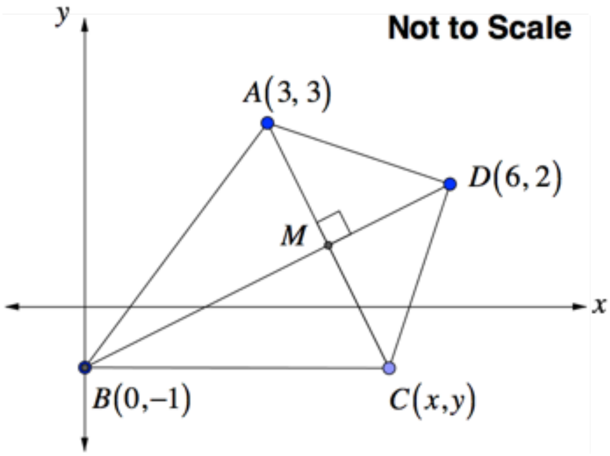
$$7xu = 3x \qquad v = x^2 + 3$$
$$u' = 3 \qquad v' = 2x$$

$$\begin{aligned} f'(x) &= \frac{vu' - uv'}{v^2} \\ \therefore f'(x) &= \frac{((x^2+3)\times 3) - (3x \times 2x)}{(x^2+3)^2} \\ &= \frac{3x^2 + 9 - 6x^2}{(x^2+3)^2} \\ &= \frac{-3(x^2-3)}{(x^2+3)^2} \end{aligned}$$

End of Question 11

Question 12

(a) In the quadrilateral $ABCD$ the points$A$, B, and D are $(3,3), (0,-1)$and$(6,2)$ respectively. The line BD bisects the line AC at right angles to the point M



(i) Find the distance BD

1 mark	Finds distance correctly

$$\begin{aligned} BD &= \sqrt{(6-0)^2 + (2-(-1))^2} \\ &= \sqrt{36+9} \\ &= \sqrt{45} \end{aligned}$$

(ii) Show that the gradient of BD is $\dfrac{1}{2}$

1 mark	Finds gradient correctly

$$\begin{aligned} m_{BC} &= \frac{2-(-1))}{6-0} \\ &= \frac{1}{2} \end{aligned}$$

(iii) Show that the equation of line BD is $x - 2y - 2 = 0$

1 mark	Uses working to show equation of line is $x - 2y - 2 = 0$

$$\begin{aligned} y-(-1) &= \frac{1}{2}(x-0) \\ \therefore y+1 &= \frac{1}{2}x \\ \therefore 2y+2 &= x \\ \therefore x-2y-2 &= 0 \end{aligned}$$

Question 12 continued

(iv) Show that the equation of AC is $2x + y - 9 = 0$

1 mark	Finds correct gradient of perpendicular line **with reasoning** $AC \perp BD$ and shows equation is $2x + y - 9 = 0$

$$\begin{aligned} m_{AC} &= -2 \qquad \text{(since } AC \perp BD\text{)} \\ \therefore y - 3 &= -2(x - 3) \\ \therefore y - 3 &= -2x + 6 \\ \therefore 2x + y - 9 &= 0 \end{aligned}$$

(v) Find the coordinates of M

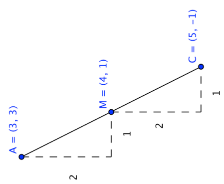
2 marks	Correctly solves the set of simultaneous equations with working to obtain $M = (4, 1)$
1 mark	Correctly sets up set of simultaneous equations but makes minor error **OR** does not state the coordinates of M

To find the coordinates of M we need to solve simultaneously$x-2y-2 = 0$ and $2x + y - 9 = 0$

$$\begin{aligned} x - 2y - 2 &= 0 \\ \therefore x &= 2y + 2 \qquad (1) \\ \text{and } 2x + y - 9 &= 0 \qquad (2) \\ \text{Substitute(1) into (2)} \\ \therefore 2(2y + 2) + y - 9 &= 0 \\ \therefore 5y &= 5 \\ \therefore y &= 1 \text{ substitute into (1)} \\ \therefore x &= 4 \\ \therefore M &= (4, 1) \end{aligned}$$

(vi) Find the coordinates of C

1 mark	Any reasoning that gives $C = (5, -1)$ or correct working for their M



$\therefore C = (5, -1)$

Question 12 continued

(b) Prove that $\dfrac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = 1 - \sin\theta\cos\theta$

2 marks	Proves the desired result with sufficient working
1 mark	Correctly factorises the numerator

$$\begin{aligned}\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} &= \frac{\cancel{(\sin\theta + \cos\theta)}(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)}{\cancel{(\sin\theta + \cos\theta)}}\\ &= \sin^2\theta - \sin\theta\cos\theta + \cos^2\theta\\ &= 1 - \sin\theta\cos\theta \quad \square \qquad (\text{since } \sin^2\theta + \cos^2\theta = 1)\end{aligned}$$

(c) If α and β are the roots of $2x^2 - 5 - 7 = 0$ find the value of $\alpha^2 + \beta^2$

2 marks	Correct answer with working
1 mark	Correct value of $\alpha + \beta$ and $\alpha\beta$ OR $(\alpha+\beta)^2 - 2\alpha\beta = \alpha^2 + \beta^2$

$$2x^2 - 5x - 7 = 0 \qquad \alpha + \beta = -\frac{5}{2} \qquad \alpha\beta = -\frac{7}{2}$$

$$\begin{aligned}\therefore \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta\\ &= \left(\frac{5}{2}\right)^2 - \times\frac{-7}{2}\\ &= \frac{25}{4} + 7\\ &= \frac{53}{4}\end{aligned}$$

Question 12 continued

(d) Differentiate with respect to x:

(i) $x^4 \sin 2x$

2 marks	Correct derivative (does not need to be factorised)
1 mark	Identifies need to use product rule

$$\begin{aligned} f(x) &= x^4 \sin 2x \\ \therefore f'x) &= 4x^3 \times \sin 2x + -x^4 \times 2\cos 2x \\ &= 2x^3(2\sin 2x + x\cos 2x) \end{aligned}$$

(ii) $(3 + \log_e x)^5$

2 marks	Correct derivative
1 mark	Identifies need to use chain rule

$$\begin{aligned} f(x) &= (3 + \log_e x)^5 \\ \therefore f'(x) &= 5(3 + \log_e x)^4 \times \frac{1}{x} \\ &= \frac{5}{x}(3 + \log_e x)^4 \end{aligned}$$

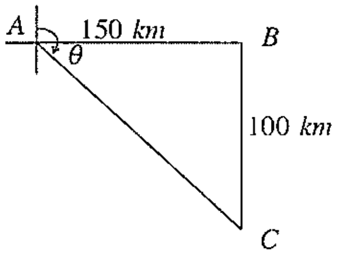
End of Question 12

Year 12 Mathematics Trial Solutions

13a)

Cinca Island is 150km due east of port A. A boat is 100km due south of Cinca Island. Calculate the bearing of the boat from port A to the nearest degree.

2 marks	obtains bearing with correct working
1 mark	gives correct expression for $\tan\theta$



$$\tan\theta = \frac{100}{150} \quad (1)$$

$$\theta = 33.69006$$

$$bearing = 90 + 34$$

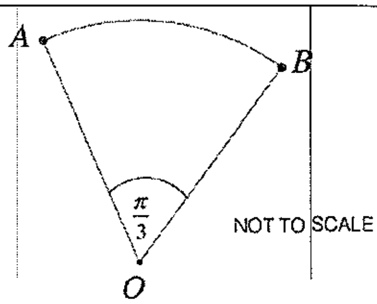
$$= 124^\circ \quad (1)$$

13b)

AOB is a sector of a circle, centre O, with angle $\frac{\pi}{3}$.

The area of the sector AOB is $\frac{25\pi}{6} cm^2$.

What is the exact length of the arc AB ?



2 marks	correct arc length as exact value with working
1 mark	value of radius with working

$$\text{Area sector} = \frac{1}{2}r^2\theta$$

$$\frac{1}{2}r^2 \times \frac{\pi}{3} = \frac{25\pi}{6}$$

$$r^2 = 25$$

$r = 5cm$ (radius is positive) (1)

$$\text{arc length } l = 5 \times \frac{\pi}{3}$$

$$= \frac{5\pi}{3} \text{ cm} \quad \text{ECF} \quad (2)$$

Q13c)

Consider the function $f(x)=2x^3-4x^2$

i) Find the coordinates of the stationary points of the curve $y=f(x)$ and determine their nature.

ii) Find the point of inflexion.

iii) Sketch the curve showing where it meets the axes.

(i)

3 marks correct co-ordinates of stationary point with working to justify max or min
2 marks correct x and y-co-ordinate of 1 stationary point with nature

1 mark correct x-co-ordinate of stationary points

$f(x)=2x^3-4x^2$

$f'(x)=6x^2-8x$

let $f'(x)=0$ then

$6x^2-8x=0$

$2x(3x-4)=0$

$x=0,\ x=\frac{4}{3}$ are stationary points

$f''(x)=12x-8$

$f''(0)=-8<0$ then maximum at (0,0) (value<0 must be shown)

$f''(\frac{4}{3})=8>0$ then a minimum at $(\frac{4}{3},-\frac{64}{27})$ (Value >0 must be shown)

(ii)

2 marks correct coordinates of point of inflection with calculations showing change in concavity
1mark coordinates of point of inflection

at point of inflection, $f''(x)=0$

$f''(x)=12x-8=0$

$x=\frac{2}{3}$ is a possible pt of inflection

$y=\frac{-32}{27}$ (1)

must show change of concavity

x	0	$\frac{2}{3}$	1
$f''(0)$	-8	0	4
	\	—	/

(values must be shown)

Concavity changes so point of inflection at $\left(\frac{2}{3},-\frac{32}{27}\right)$ (2)

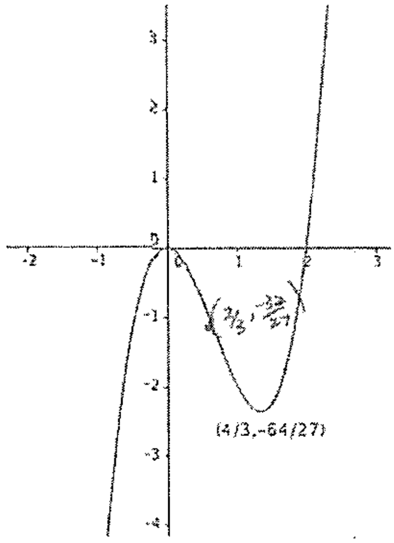
2 marks correct graph with intercepts , point of inflection and turning points shown
1 mark correct graph with 1 requirement no shown or incorrect

Find x-intercepts

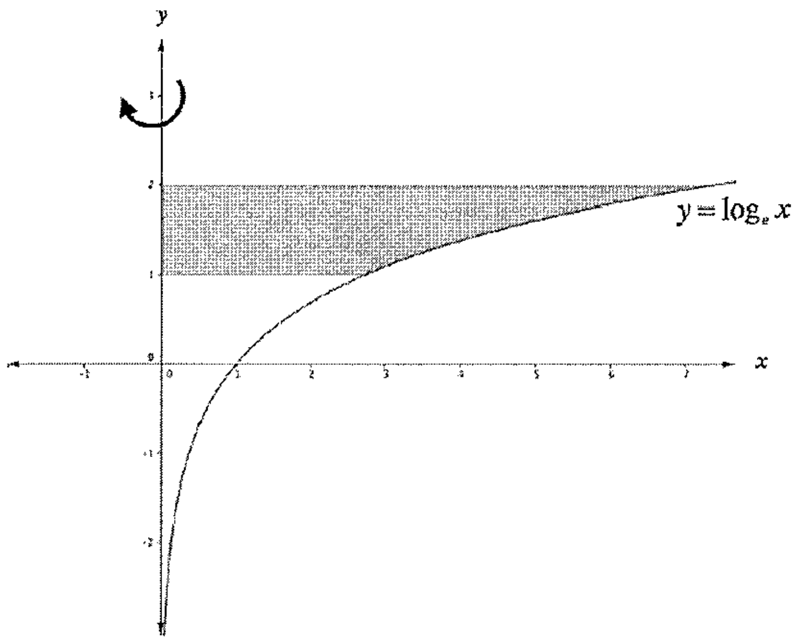
$f(x)=0$ when $2x^3-4x^2=0$

$2x^2(x-2)=0$

$x=0,\ 2$



Q13d) The curve $y=\log_e x$ is shown below.



i) Show that when x is made the subject of the equation, the equation of the curve is $x=e^y$.

1 mark correct expression

$y=\log_e x$

$e^y=e^{\log_e x}$

$\therefore x=e^y$

OR quote log laws

$\log_b x=a$

$\therefore b^a=x$

$\therefore \log_e x=y$

$e^y=x$

ii) In the diagram the shaded region is bounded by the curve $y = \log_e x$, the y-axis and the lines $y = 1$ and $y = 2$

The shaded region is rotated about the y-axis.

Calculate the exact volume of the solid of revolution formed.

3 marks	correct calculation of volume with working as an exact value
2 marks	correct integration
1 mark	correct expression for integral

$$V = \pi\int_1^2 x^2 dy$$

$$= \pi\int_1^2 e^{2y} dy \qquad (1)$$

only 1 mark if e^y as made clear earlier.

$$= \pi\left[\frac{e^{2y}}{2}\right]_1^2 \qquad (2)$$

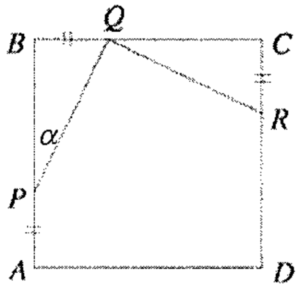
$$= \pi\left[\frac{e^4}{2} - \frac{e^2}{2}\right]$$

$$= \frac{\pi}{2}(e^4 - e^2) \qquad (3)$$

ECF or $\frac{\pi}{2}e^2(e^2-1)$

14a)	(i) Solve the following: $2^{2x} - 12(2^x) + 32 = 0$ 3 marks correct values with working shown 2 marks correct solving for 1 mark correct factorising $(2^x - 4)(2^x - 8)$ $2^x = 4$ or $2^x = 8$ $2^x = 2^2$ or $2^x = 2^3$ $x = 2$ or $x = 3$

b) *ABCD* is a square. The points *P*, *Q* and *R* lie on *AB*, *BC* and *CD* respectively so that *AP*=*BQ*=*CR*.



α

i) Prove that $\Delta PBQ \equiv \Delta QCR$.

(i)

2 marks correct proof with full reasoning and conclusion 1 mark 2 correct statements with reasoning

)

In ΔPBQ and ΔQCR

$BQ = RC$ (given)

$\angle PBQ = \angle RCQ = 90^o$ (angle in a square)

$AB = BC$ (equal sides of a square)

$AP = CR$ (given)

$\therefore \Delta PBQ \equiv \Delta QCR$ (SAS)

ii) Prove that *PQ* is perpendicular to QR.

2 marks correct proof with full reasoning 1 mark working towards solution with correct reasoning

Let $\angle QPB = \alpha$

then $\angle RQC = \alpha$ (corresponding angle in congruent triangle equal)

$\angle BQP = 90 - \alpha$ (complementary angle in right angled triangle)

similarity $\angle CRQ = 90 - \alpha$ (complementary angle in right angled triangle)

then $\angle PQR = 180 - (90 - \alpha) - \alpha$ (angle on a straight line)

$= 180 - 90 + \alpha - \alpha$

$= 90^o$

c)	A particle moving in a straight line is initially at the origin. The displacement, in metres, after t seconds is given by $x = 2t - 3\log_e(t+1)$.

i) Find an expression for the velocity.

ii) Find the initial velocity.

iii) Find when the particle is at rest and its position at this time (answer correct to 2 decimal places)

2 marks correct expression for velocity 1 mark one error in expression for velocity

$$x = 2t - 3\log_e(t+1)$$

$$\frac{dx}{dt} = 2 - \frac{3}{t+1}$$

1 mark correct value for velocity

ii) Find the initial velocity.

$$v = 2 - \frac{3}{t+1}$$

for initial velocity $t = 0$

$$v = 2 - 3$$

$$v = -1 \text{ m/s}$$

iii) Find when the particle is at rest and its position at this time (answer correct to 2 decimal places)

1 mark correct calculation of time with working 1 mark correct position with working

particle at rest when $v = 0$

$$x' = 2 - \frac{3}{t+1} = 0$$

$$2 = \frac{3}{t+1}$$

$$2t + 2 = 3$$

$$2t = 1$$

$$t = \frac{1}{2} \text{ sec} \quad (1)$$

$$x = 2 \times \frac{1}{2} - 3\log_e(1.5)$$

$$= 1 - 3\log_e(1.5)$$

$$x = -0.216 \text{ m} \quad (2)$$

d)

There are five candidates, Allan, Brown, Chin, Davis and Echert standing for the seat of Bradfield in the federal election. Their names are written on pieces of paper and randomly drawn from a barrel to determine their positions on the ballot paper. The candidate picked first goes at the top of the list

i) What is the probability that Davis is drawn first?

1 mark correct answer

$$P(D)=\frac{1}{5}$$

ii) What is the probability that the order the names appear on the ballot paper is as follows.

Allan	
Brown	
Chin	
Davis	
Echert	

2 marks correct answer with working shown
1 marks working towards correct answer with working

$$P(D)=\frac{1}{5}\times\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}$$
$$=\frac{1}{240}$$

Comments on Q15.

15a)	Crumponium is a rare radioactive substance that decays with a highly toxic residue. The rate of change is given by $$\frac{dM}{dt}=-kM$$ where k is a positive constant and M is the mass present.

i) The half-life of Crumponium is 29 years. This means it takes 29 years for 100g to decay to 50g. Find the value of k correct to 3 significant figures.

2 marks	correct value with calculations
1 mark	correct integral with substitution

(i)

$$\frac{dM}{dt}=-kM$$

$$M=M_o e^{-kt}$$

$$\frac{1}{2}=e^{-k\times 29} \qquad (1)$$

$$\log_e(0.5)=-29k$$

$$k=0.0239 \qquad (2)$$

Some students gave very long solutions which included deriving $M=M_0e^{-kt}$ this was not necessary.

There was no deduction this time for incorrect rounding, but many students made an error rounding to 3 significant figures.

ii) A decaying bag of Crumponium is found illegally dumped at a landfill site. It is weighed and its mass is 12 kg. Calculate the original mass if it was dumped 10 years ago. Give your answer to 2 decimal places.

1 mark correct answer with working

$$12=M_o e^{-0.0239\times 10}$$

$$M_o=\frac{12}{e^{-0.239}}$$

$$M_o=15.2399....$$

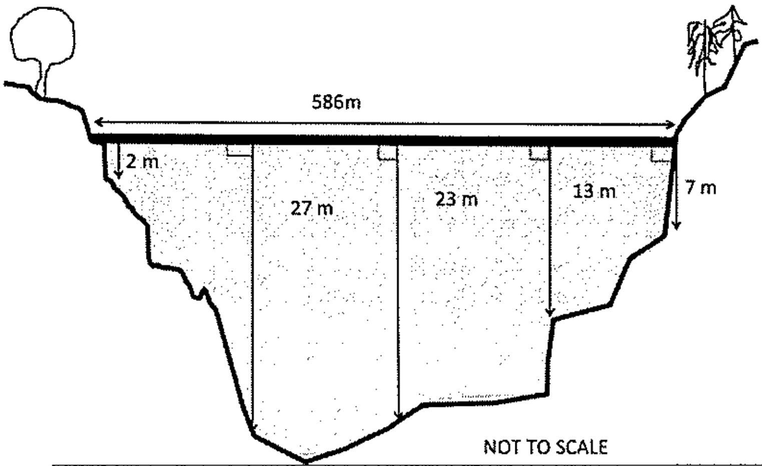
$$M_o=15.24\text{ kg}$$

Done well by students who completed Part (i)

b) Engineers have recorded the average depth measurements for a new dam at Jindabyne.

The diagram below shows the cross sectional area along the dam wall. The depth measurements are taken at equidistant points along the dam wall. The diagram is not to scale.

Use Simpsons Rule to determine the cross sectional area of the dam wall. Give your answer to the nearest square metre. **2**



2 marks	correct calculation of area with working
1 mark	correct expression for area with substitution or answer with 1 error

1 mark

$$A=\frac{h}{3}\left[f_o+4f_1+2f_2+......+4f_{n-1}+f_n\right]$$

$$=\frac{586}{12}\left[2+4\times 27+2\times 23+4\times 13+7\right] \quad (1)$$

$$A=10499.16....$$

$$A=10499\text{ m}^2 \quad (2)$$

Mostly done well.
Many students made an error calculation. 'h'.
4 Subintervals means $h=\frac{586}{4}$

1 mark was allocated to the correct function values in the brackets with correct multipliers.

c)

For what values of k will the expression $kx^2 + 2x + k$ always be negative? **2**

Correct calculation for k
Correct expression for discriminant with working

$kx^2 + 2x + k$ is always negative for $k < 0$ and $\Delta < 0$

$\Delta = b^2 - 4ac < 0$

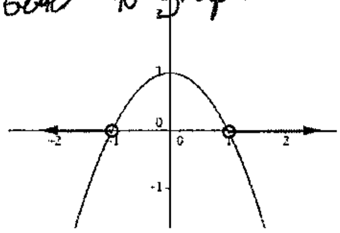
$4 - 4k^2 < 0$ (1)

$4(1+k)(1-k) < 0$

$k > 1$ and $k < -1$ but $k < 0$

$\therefore\ k < -1$ (2)

Many students incorrectly solved the inequality. Much better to graph it.
For positive definite
$k<0,\ \Delta<0$



15d)

Show that the locus of a point that moves so that its distance from the point A $(-5, 2)$ is twice its distance from the point B $(1, 2)$ is a circle with centre $(3, 2)$ and Radius $r = 4$ **3**

3 marks correct working to show centre and radius of circle
2 marks simplified expression
1 mark correct simplified expression for centre and radius

distance from A = 2 × distance from B

$$d_A{}^2 = 4d_B{}^2$$

$$(x+5)^2 + (y-2)^2 = 4\left[(x-1)^2 + (y-2)^2\right] \quad (1)$$

$$x^2 + 10x + 25 + y^2 - 4y + 4 = 4(x^2 - 2x + 1 + y^2 - 4y + 4)$$

$$x^2 + 10x + y^2 - 4y + 29 = 4x^2 - 8x + 4 + 4y^2 - 16y + 16$$

$$3x^2 - 18x + 3y^2 - 12y - 9 = 0 \quad (2)$$

$$x^2 - 6x + y^2 - 4y - 3 = 0$$

$$(x^2 - 6x + 9) + (y^2 - 4y + 4) = 3 + 9 + 4 \quad (3)$$

$$(x-3)^2 + (y-2)^2 = 16$$

equation circle centre (3,2) radius = 4

A common error was $4d_A{}^2 = d_B{}^2$. Students who made this error lost 1 mark if they showed the rest of their working correctly.

Too many students showed the final line $(x-3)^2+(y-2)^2=16$ with incorrect working & lost marks. Please don't fudge your working!

15e)	A circular barbeque plate is being heated so the rate of increase of the area $A\,cm^2$ after t minutes is given by $$\frac{dA}{dt}=\frac{\pi}{10(t+1)}$$ The plate has an initial Area of 45 cm^2. Find the area of the plate after it has been heated for 50 minutes (give your answer correct to 2 decimal places)	3

3 marks	correct area with working
2 marks	correct calculation of constant c
1 mark	correct expression for integral

$$\frac{dA}{dt}=\frac{\pi}{10(t+1)}$$

$$A=\int\frac{\pi}{10}\times\frac{1}{t+1}dt$$

$$=\frac{\pi}{10}\int\frac{1}{t+1}dt$$

$$=\frac{\pi}{10}\ln(x+1)+c \qquad (1)$$

when $t=0$ initial Area $A=45$

$$A=\frac{\pi}{10}\ln(0+1)+c=45$$

$$\ln 1=0$$

$$0+c=45$$

$$c=45 \qquad (2)$$

then $A=\frac{\pi}{10}\ln(x+1)+45$

when $t=50$

$$A=\frac{\pi}{10}\ln(50+1)+45$$

$$=46.2352.....$$

$$=46.24 \text{ (2dp)} \qquad (3)$$

Common errors were

- Incorrect integral $\int\frac{1}{t+1}=\ln(x+1)+c$
- Not calculating the constant.

The simplest solution is shown. An alternate but more complicated solution is shown below.

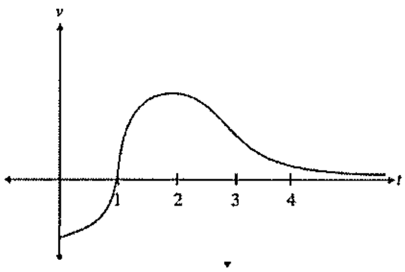
$$A=\frac{\pi}{10}\int\frac{10}{10t+10}\,dt$$

$$=\frac{\pi}{10}\left[\ln(10t+10)\right]+c$$

$$c=45-\frac{\pi\ln 10}{10}$$

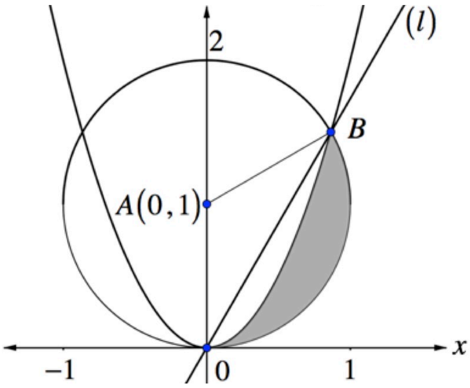
$$\therefore A=\frac{\pi}{10}\left[\ln(10t+10)\right]+45-\frac{\pi\ln 10}{10}$$

$$=46.24$$

f)	A particle moves along the x-axis. Its velocity v ms^{-1} after t seconds is shown in the diagram  Done well. i) Initially is the particle moving to the left or the right. Explain your answer. 1 mark correct answer with valid explanation initial velocity<0 so particle is moving left
	How would you calculate the distance the particle travels in the first 3 seconds? 1 mark correct expression or explanation including absolute value of negative area total distance is the integral of the velocity Total distance is the absolute value of the area under the curve distance $= \left\|\int_0^1 v\,dt\right\| + \int_1^3 v\,dt$ Common error was not stating the absolute value. A short explanation with expression is all that is required.

Question 16

(a) The circle centered at A with radius 1 unit intersects the parabola $y = x^2$ at the origin 0 and the point B. The line l passes through 0 and B as shown in the diagram.



(i) Show that the coordinates of B are $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$

2 marks	Solves simultaneous equations correctly to obtained desired result
1 mark	Minor error or finds x coordinate or y coordinate only

The equation of the circle is $x^2 + (y-1)^2 = 1$ (1)

this needs to be solved simultaneously with $y = 2x^2$ to find B

$$\therefore x^2 = \frac{y}{2} \quad (2)$$

Substitute (2) into (1)

$$\therefore \frac{1}{2}y + y^2 - 2y + 1 = 1$$

$$\therefore y^2 - \frac{3}{2}y = 0$$

$$\therefore 2y^2 - 3y = 0$$

$$\therefore y(2y-3) = 0$$

$$\therefore y = 0, \frac{3}{2} \text{ but } y \neq 0$$

$$\therefore y = \frac{3}{2} \text{ substitute this into (2)}$$

$$\therefore x^2 = \frac{3}{4}$$

$$\therefore x = \pm\frac{\sqrt{3}}{2} \text{ but } x > 0$$

$$\therefore x = \frac{\sqrt{3}}{2} \therefore B = \left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$$

Question 16 continued

(ii) Find the angle OB makes with the positive x axis

1 mark	Correct answer with working

$$m_{OB} = \frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\therefore \tan\alpha = \sqrt{3} \therefore \alpha = \frac{\pi}{3} \text{ since } \alpha \text{ is actute}$$

(iii) Show that $\angle AOB$ is $\dfrac{2\pi}{3}$

2 marks	Shows desired result with all reasons]
1 mark	Shows desired result with only some of the reasons i.e leaves out a reason (but has got some reasoning

$$\angle AOB = \frac{\pi}{2} - \frac{\pi}{3} \qquad \text{(complementary adjacent:angles)}$$

$$\therefore \angle AOB = \frac{\pi}{6}$$

$$AO = AB = 1 \qquad \text{(equal radii)}$$

$$\therefore \angle AOB = \angle ABO = \frac{\pi}{6} \qquad \text{(equal angles opposite equal sides in an isosceles triangle)}$$

$$\therefore \angle OAB = \pi - \frac{\pi}{6} - \frac{\pi}{6} \qquad \text{(angle sum of triangle } OAB\text{)}$$

$$= \frac{2\pi}{3} \quad \square$$

Question 16 continued

(iv) Find the shaded area bounded by the circle and the parabola in the first quadrant as shown in the diagram

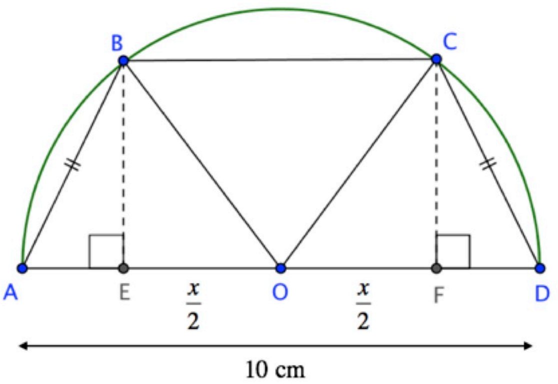
To find the shaded find the area between l and $y = 2x^2$ and subtract it from the area of the minor segment

$$\begin{aligned} \text{Area of segment } &= \frac{1}{2}r^2(\theta - \sin\theta) \\ \therefore A_1 &= \frac{1}{2}\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right) \\ &= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) \end{aligned}$$

Area between two curves $y = \sqrt{3}x$ and $y = 2x^2$:

$$\begin{aligned} A_2 &= \int_0^{\frac{\sqrt{3}}{2}} \left(\sqrt{3}x - 2x^2\right)\, dx \\ &= \left[\frac{\sqrt{3}}{2}x^2 - \frac{2}{3}x^3\right]_0^{\frac{\sqrt{3}}{2}} \\ &= \left[\frac{3\sqrt{3}}{8} - \frac{6\sqrt{3}}{24}\right] = \frac{\sqrt{3}}{8}\text{units}^2 \\ \therefore \text{Shaded area } &= A_1 - A_2 \\ &= \frac{\pi}{3} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{8} = \left(\frac{\pi}{3} - \frac{3\sqrt{3}}{8}\right)\text{ units}^2 \end{aligned}$$

(b) An isosceles trapezium $ABCD$ is drawn with tis vertices on a semicircle centre O and diameter 10 cm



(i) If $EO = OF = \frac{x}{2}$ show that $BE = \frac{1}{2}\sqrt{100 - x^2}$

2 marks	Shows desired result with reasons
1 mark	Shows desired result but misses a reason

$$\begin{aligned} BE^2 &= 5^2 - \left(\frac{x}{2}\right)^2 && \text{(by Pythagoras')} \\ &= 25 - \frac{x^2}{4} \\ &= \frac{1}{4}\left(100 - x^2\right) \\ \therefore BE &= \frac{1}{2}\sqrt{100 - x^2} \quad \Box && \text{(only positive result as } BE \text{ is a length)} \end{aligned}$$

(ii) Show that the area of the trapezium is$ABCD$ is give by

$$A = \frac{1}{4}(x + 10)\sqrt{100 - x^2}$$

2 marks	Shows desired result with sufficient working
1 mark	Substitutes correctly into formula for trapezium or makes minor error with trapezium formula

The area of a trapezium is given by$A = \frac{1}{2}(a + b) \times h$

$$\begin{aligned} \therefore A &= \frac{1}{2}\left(10 + \left(\frac{x}{2} + \frac{x}{2}\right)\right) \times \frac{1}{2}\sqrt{100 - x^2} \\ &= \frac{1}{4}(x + 10)\sqrt{100 - x^2} \quad \Box \end{aligned}$$

Question 16 continued

(iii) Hence find the length of BC so that the area of the trapezium is a maximum

3 marks	Finds correct length of BC showing full working and reasoning
2 marks	Finds correct length of BC but does not show it is a maximum (must give reason for excluding negative value of x)
1 mark	Correctly finds $A'(x)$ (doesn't need to be factorised)

$$A(x) = \frac{1}{4}(x+10)\sqrt{100-x^2} = \frac{1}{4}(x+10)(100-x^2)^{\frac{1}{2}}$$

$$\therefore A'(x) = \left(\frac{x+10}{4}\right)\left(\frac{-x}{\sqrt{100-x^2}}\right) + \frac{\sqrt{100-x^2}}{4}$$

$$= \frac{-x(x+10)+100-x^2}{4\sqrt{100-x^2}}$$

$$= \frac{-2x^2-10x+100}{4\sqrt{100-x^2}}$$

$$= \frac{-(x^2-5x+50)}{2\sqrt{100-x^2}}$$

$$= \frac{-(x-5)(x+10))}{2\sqrt{100-x^2}}$$

For a maximum $A'(x) = 0$

$\therefore (x-5)(x+10) = 0$

$\therefore x = 5$ (since $x > 0$ as it is a length)

Need to test whether the function is a maximum at $x = 5$

x	4	5	4
$f'(x)$	0.76	0	−1

As the derivative changes sign at $x = 5$ then there is a local maximum at $x = 5$

$\therefore$ Maximum area of trapezium occurs when $BC = \frac{5}{2} + \frac{5}{2} = 5$ cm

End of Question 16